Phase Noise in Self-Injection-Locked Oscillators—Theory and Experiment

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***Abstract*—Phase-noise analysis of the self-injection-locked oscillator is presented in this paper. The analysis is developed for different oscillator models and arbitrary self-injection feedback loops. The results are illustrated with specific cases of simple time-delay cable and a high-Q factor resonator. It is shown that the behavior of the phase noise is similar to an oscillator locked to an external low phase-noise source. The output phase noise can be reduced at the noise offset frequency near the carrier frequency, and returning to the free-running oscillator noise far from the carrier frequency for certain stable feedback delay ranges. The phase-noise reduction is affected by the self-injection signal strength and feedback transfer function for different oscillator equivalent-circuit models. The theory is verified by using a self-injection-locked GaAs MESFET oscillator operating at the X-band with delay cable loops. The self-injection-locked technique may be used to improve the phase noise of the existing oscillators.**

***Index Terms*—AM noise, delay line, feedback loop, injection lock, noise, oscillator, parallel resonant, phase (PM) noise, resonator, self-injection lock, series resonant, stability.**

I. Introduction

COMMERCIAL and military applications for microwave and millimeter-wave sources in communication systems require low phase-noise oscillators. Similarly, commercial digital communication systems also put strict constraints on the signal-to-noise ratio and bit error rate (BER) for high-fidelity information transmission. Phase-noise reduction in the oscillators typically use the very high-Q external resonator in the oscillator circuits [1] or pass the oscillator output signal through the high-Q cavity to eliminate the noise components. Injection locking the oscillator [2], [3] with an external low phase-noise source has been proposed to reduce the oscillator phase noises [4]–[6]. The noise reduction in single one- and coupled-oscillator arrays phase locked to the external low phase-noise signal has been verified in the authors' previous work by theories and experiments [7]–[9].

A self-injection lock technique has been used in lasers to reduce the linewidth and frequency (or phase) noise [10], [11]. A part of the oscillator output signal is used to injection lock the oscillator itself. The self-injection signal has the same frequency as the oscillator, and it is easy for the oscillator to remain phase locked for certain stability conditions [12]–[14]. The stability analysis has been derived in [14]. In this paper, the author extends the previous work to explore the spectral characteristics of the noise and the dependence on the feedback transfer function and self-injection signal strength in theory and verify the results by experiments. The specific cases of the simple cable delay line and the high-Q factor resonator in the feedback loop for different oscillator models [13], [14] are analyzed in this paper. Here, we only consider the oscillator phase noise. Amplitude noise and AM-to-PM noise conversion are assumed negligibly small, as compared to PM noise. The self-injection-locked technique may be used to improve the phase noise of the existing oscillators.

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II. Phase Dynamics of a Single Noisy Oscillator With a Noisy Self-Injection Signal

The phase dynamics of the injection-locked oscillator depends on the coupling phase and the types of the oscillator circuits, which can be represented by an equivalent parallel- or series-resonant circuit model [13], [14]. If a part of the oscillator output signal is extracted and fed through a feedback with the transfer function [the frequency response is $H(\omega)$ and the time-domain response is $h(t)$] and then into the oscillator injection port (see [14, Fig. 1]), the phase relationship of the self-injection-locked oscillator [13], [14] becomes

$$\frac{d\theta(t)}{dt} = \omega_0 \pm \rho\omega_{3\mathrm{dB}} \sin\left(\theta_{\mathrm{inj}}(t) - \theta(t)\right) - \omega_{3\mathrm{dB}} B_n(t) \quad (1)$$

where ω_0 is the carrier frequency, $\theta(t)$, $\theta_{\mathrm{inj}}(t) = h(t) * \theta(t)$ are the instantaneous phases of the oscillator output signal and the injection signal, respectively, and $(*)$ is the convolution symbol. ω_0 and Q are the free-running frequency and Q factor of the oscillator, respectively. $\omega_{3\mathrm{dB}} = \omega_0/(2Q)$ is half the 3-dB bandwidth of the oscillator tank circuits. The upper sign is for parallel-resonant oscillators and the lower sign is for series-resonant oscillators. The coupling phase Φ [7], [8] or delay from the injection signal source has been included in $\theta_{\mathrm{inj}}(t)$. $\rho = A_{\mathrm{inj}}/A$ is the injection strength, and the injection signal A_{inj} is normalized to the oscillator's free-running amplitude A. $B_n(t)$ is a time-varying noise susceptance (or the quadrature phase component of the noise admittance) and is assumed to be an ergodic process [7], [8].

A steady-state noise-free synchronized state for the oscillators satisfies

$$\sin\left(\hat{\theta}_{\mathrm{inj}}(t) - \hat{\theta}(t)\right) = \pm\frac{(\omega_{\mathrm{inj}} - \omega_0)}{\rho\omega_{3\mathrm{dB}}} = \pm\frac{(\omega_{\mathrm{inj}} - \omega_0)}{\Delta\omega_{\mathrm{lock}}} \quad (2)$$

where $d\theta/dt = \omega_{\mathrm{inj}}$ is the injection frequency, $\omega_{3\mathrm{dB}} = \omega_0/(2Q)$ is half the 3-dB bandwidth of the oscillator tank circuits, $\Delta\omega_{\mathrm{lock}} = \rho\omega_{3\mathrm{dB}}$ is half the entire locking range, and the circumflex $(\hat{\ })$ denotes a steady-state quantity. The upper sign is for parallel oscillators and the lower sign is for series oscillators.

The differential equation (1) without the $B_n(t)$ term is an implicit equation of $\theta(t)$, and one needs to use the numerical method to find the exact steady-state solution with the initial value θ_0. However, the locking range $\rho\omega_{3\text{dB}} \ll \omega_0$ in real self-injection-locked oscillators and the steady-state phase can be approximated by $\theta(t) \approx \omega_0 t + \theta_0$.

III. Phase Noise of the Self-Injection-Locked Oscillators

After knowing the phase dynamics of the self-injection-locked oscillator, one wants to find the phase-noise expressions. The phasor expression of the injection signal is

$$F\left[e^{\jmath\theta_{\text{inj}}(t)}\right] = H(\omega)F\left[e^{\jmath\theta(t)}\right] \tag{3}$$

where F denotes the Fourier transform and $H(\omega)$ is the self-injection-locked feedback transfer function in the frequency domain, as shown in [14, Fig. 1]. Assuming the noise is a small perturbation to a noise-free solution, one can write $\theta(t) \to \hat{\theta}(t) + \delta\theta(t) = \omega_0 t + \delta\theta(t)$, $\theta_{\text{inj}}(t) \to \hat{\theta}_{\text{inj}}(t) + \delta\theta_{\text{inj}}(t)$, $e^{\jmath\delta\theta(t)} \approx 1 + \jmath\delta\theta(t)$, and $e^{\jmath\delta\theta_{\text{inj}}(t)} \approx 1 + \jmath\delta\theta_{\text{inj}}(t)$ in (1) and (3), where $\delta\theta(t)$ and $\delta\theta_{\text{inj}}(t)$ describe the small phase fluctuations of the oscillator output and injection signals, respectively. Equation (3) becomes

$$\begin{aligned} F\left[e^{\jmath\theta_{\text{inj}}(t)}\right] &= F\left[e^{\jmath\left(\hat{\theta}_{\text{inj}}(t)+\delta\theta_{\text{inj}}(t)\right)}\right] \\ &\approx F\left[e^{\jmath\hat{\theta}_{\text{inj}}(t)}\right] + \jmath F\left[e^{\jmath\hat{\theta}_{\text{inj}}(t)}\delta\theta_{\text{inj}}(t)\right] \\ &= H(\omega)F\left[e^{\jmath(\omega_0 t+\delta\theta(t))}\right] \\ &\approx H(\omega)\left(F[e^{\jmath\omega_0 t}] + \jmath F\left[e^{\jmath\omega_0 t}\delta\theta(t)\right]\right) \\ &= H(\omega)\delta(\omega-\omega_0) + \jmath H(\omega)\widetilde{\delta\theta}(\omega-\omega_0) \end{aligned} \tag{4}$$

where $\delta(\omega)$ is the Dirac delta function. The output of the feedback loop in the self-injection-locked oscillator has the same frequency as the oscillator output so that the phase fluctuation of the injection signal around the carrier frequency ω_0 in the frequency domain is

$$F\left[e^{\jmath\hat{\theta}_{\text{inj}}(t)}\delta\theta_{\text{inj}}(t)\right] = \widetilde{\delta\theta}_{\text{inj}}(\omega-\omega_0) = H(\omega)\widetilde{\delta\theta}(\omega-\omega_0) \tag{5}$$

(or $\widetilde{\delta\theta}_{\text{inj}}(\omega) = H(\omega+\omega_0)\widetilde{\delta\theta}(\omega)$) and the relationship in the time domain is

$$\delta\theta_{\text{inj}}(t) = \left(h(t)e^{-\jmath\omega_0 t}\right) * \delta\theta(t) \tag{6}$$

where $(*)$ is the convolution symbol.

Therefore, for self-injection-locked oscillators the phase fluctuation (1) becomes

$$\frac{d\delta\theta(t)}{dt} = \pm\rho\omega_{3\text{dB}}\cos\Delta\hat{\theta}_p\left[\left(h(t)e^{-\jmath\omega_0 t}\right) * \delta\theta(t) - \delta\theta(t)\right] - \omega_{3\text{dB}}B_n(t) \tag{7}$$

where the upper sign is for parallel oscillators and the lower sign is for series oscillators.

The loop phase can be defined as

$$\Delta\hat{\theta} = \hat{\theta}(t) - \hat{\theta}(t-T). \tag{8}$$

If $\Delta\hat{\theta}$ is independent of a time variable (i.e., t), Fourier transforming (7) and rearranging terms gives

$$\widetilde{\delta\theta}(\omega_m) = \frac{-\omega_{3\text{dB}}\widetilde{B}_n(\omega_m)}{\jmath\omega_m \mp \rho\omega_{3\text{dB}}\cos\Delta\hat{\theta}\left(H(\omega_0+\omega_m)-1\right)} \tag{9}$$

where the tilde $(\widetilde{\ })$ denotes a transformed or spectral variable. ω_0 is the carrier frequency and ω_m is the noise offset frequency from the carrier frequency. ω_m is used as the noise offset frequency instead of ω for the clarity of symbol notations. The upper sign is for parallel oscillators and the lower sign is for series oscillators. The power spectrum of the oscillator phase fluctuation is calculated from $\langle\widetilde{\delta\theta}(\omega_m)\widetilde{\delta\theta}^*(\omega_m)\rangle$, where the notation $\langle\ \rangle$ represents an ensemble average [7], [8].

In the absence of the self-injection signal ($\rho = 0$), the convolution terms of $h(t)$ in (7) and (9) disappear and the power spectrum density of the oscillator phase fluctuation (the phase noise) reduces to the familiar phase fluctuation spectral density of a single one free-running oscillator [7], [8]

$$\left|\widetilde{\delta\theta}(\omega_m)\right|^2 \to \left|\widetilde{\delta\theta}_0(\omega_m)\right|^2 = \left|\widetilde{\delta\theta}_i(\omega_m)\right|^2_{\text{uncoupled}} = \frac{\left|\widetilde{B}_n(\omega_m)\right|^2}{\left(\dfrac{\omega_m}{\omega_{3\text{dB}}}\right)^2} \tag{10}$$

where the $\langle\ \rangle$ notation has been dropped, an ensemble or time average being implicitly understood.

One can calculate this power spectrum by using (9) and obtain the phase noise in self-injection-locked parallel-resonant oscillators with a feedback transfer function $H(\omega)$

$$\left|\widetilde{\delta\theta}(\omega_m)\right|^2 = \frac{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2}{\left|\jmath \pm \rho\left(\dfrac{\omega_{3\text{dB}}}{\omega_m}\right)\cos\Delta\hat{\theta} \mp \rho\left(\dfrac{\omega_{3\text{dB}}}{\omega_m}\right)\left(\cos\Delta\hat{\theta}\right)H(\omega_0+\omega_m)\right|^2} \tag{11}$$

where the upper sign is for parallel oscillators and the lower sign is for series oscillators. From the stability analysis [14], the loop phase is required to satisfy $\cos\Delta\hat{\theta} > 0$ for a stable output phase. One can calculate the phase noise at certain loop phase points to illustrate the phase-noise reduction in self-injection-locked oscillators.

Now the phase-noise expressions have been obtained for self-injection-locked parallel and series-resonant oscillators with an arbitrary feedback transfer function $H(\omega)$. In the following, the specific cases of the simple delay-line cable and high-Q factor resonator in the feedback loop are illustrated to study the phase-noise performances of the oscillators.

IV. Delay-Line Cable in Feedback Loop

If the self-injection signal is fed into some cable delay line and then injection locked into the oscillator, one can assume that the cable loss is very small and negligible and $H(\omega) = e^{-\jmath\omega T}$

[and then $h(t) = \delta(t-T)$] within the interested frequency range for simplicity, where T is the loop delay time.

With (11), the phase fluctuation of self-injection-locked oscillator with a delay-line cable in the feedback loop is

$$\frac{\left|\widetilde{\delta\theta}(\omega_m)\right|^2}{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2} = \left[1 \pm 2\rho\omega_{3\mathrm{dB}}\cos\omega_0 T\left(\frac{\sin(\omega_0+\omega_m)T}{\omega_m}\right) + 2\rho^2\omega_{3\mathrm{dB}}^2\cos^2\omega_0 T\left(\frac{1-\cos(\omega_0+\omega_m)T}{\omega_m^2}\right)\right]^{-1} \tag{12}$$

where the loop phase $\Delta\hat{\theta}_p = \hat{\theta}(t) - \hat{\theta}(t-T) = \omega_0 T$ and the $\langle\,\rangle$ notation has been dropped, with an ensemble or time average being implicitly understood. The upper sign is for parallel oscillators and the lower sign is for series oscillators.

A. Loop Phase $\Delta\hat{\theta} \approx 0$ or 2π

If the loop phase $\Delta\hat{\theta} = \omega_0 T \approx 2m\pi$, where m is an integer, the phase noise (12) of the self-injection-locked parallel oscillator becomes

$$\left|\widetilde{\delta\theta}(\omega_m)\right|^2 = \frac{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2}{1 + 2\rho\omega_{3\mathrm{dB}}\dfrac{\sin\omega_m T}{\omega_m} + 2\rho^2\omega_{3\mathrm{dB}}^2\left(\dfrac{1-\cos\omega_m T}{\omega_m^2}\right)}. \tag{13}$$

By substituting

$$\lim_{\omega_m\to 0}\frac{\sin\omega_m T}{\omega_m} = T$$
$$\lim_{\omega_m\to 0}\frac{1-\cos(\omega_m T)}{\omega_m^2} = \frac{T^2}{2} \tag{14}$$

into (13), where the l'Hôpital rule is used from standard calculus, one can find the phase noise near the carrier frequency (i.e., $\omega_m \to 0$)

$$\lim_{\omega_m\to 0}\left|\widetilde{\delta\theta}\right|^2 \to \frac{\left|\widetilde{\delta\theta}_0\right|^2}{(1+\rho\omega_{3\mathrm{dB}}T)^2}. \tag{15}$$

If one increases the self-injection signal strength ρ and the loop delay T while keeping the loop phase $\Delta\hat{\theta} = \omega_0 T \approx 0$ or 2π, the phase noise of the oscillator at the noise offset frequency near the carrier frequency can be reduced further. For a longer delay-line cable, the phase noise in the self-injection (or delayed) signal will be less correlated with the undelayed signal and the related phase noise at the output port. Thus, the noise will tend to add incoherently, whereas the signal adds coherently if the delay is within the stable delay ranges for parallel- or series-resonant oscillators, as shown in [14, Fig. 3]. Therefore, the total oscillator phase noise is reduced in the feedback loop system (i.e., the self-injection-locked oscillator). However, the longer loop delay T requires the longer cable delay line, and it will be impractical to use a long cable in the self-injection-locked oscillator to reduce the phase noise.

At the other extreme (far from the carrier frequency) and $\omega_m \to \infty$, the oscillator phase noise approaches the free-running noise properties

$$\lim_{\omega_m\to\infty}\left|\widetilde{\delta\theta}\right|^2 \to \left|\widetilde{\delta\theta}_0\right|^2 \tag{16}$$

and the self-injection lock has no effect on the phase-noise reduction.

However, from the stability analysis, the phase of the series-resonant oscillators with a self-injection signal cannot be stable. The phase noise cannot be reduced by using a self-injection-locking technique under this condition.

B. Loop Phase $\Delta\hat{\theta} \approx \pi$

If the loop phase $\Delta\hat{\theta} = \omega_0 T \approx (2m+1)\pi$, where m is an integer, from the stability analysis, the phase of the parallel-resonant oscillators with a self-injection signal cannot be stable. The phase noise cannot be reduced by using a self-injection-locking technique under this condition.

For series oscillators, the phase noise (12) also becomes (13), which is the same as the self-injection-locked parallel-resonant oscillators with loop phase $\Delta\hat{\theta} = 0$ or 2π. They both have same phase-noise performances.

C. Loop Phase $\Delta\hat{\theta} \approx \pi/2$ or $3\pi/2$

If the loop phase $\Delta\hat{\theta} = \omega_0 T \approx (2m+1)\pi/2$, where m is an integer, the phase noise (12) of the self-injection-locked oscillator becomes

$$\left|\widetilde{\delta\theta}(\omega_m)\right|^2 = \left|\widetilde{\delta\theta}_0(\omega_m)\right|^2. \tag{17}$$

The injection signal has a quadrature phase delay with respect to the oscillator undelayed output signal, and then, there is no effect on phase-noise reduction for both parallel and series oscillators.

In the above derivations, we assume that there is no attenuation loss in the delay-line cables. However, there is some attenuation loss in real delay-line cables. The self-injection signal strength ρ will decrease further as the delay-line cable gets longer. One can substitute $\rho = f(\alpha, L)$ into (12) and get its derivative with respect to the delay-line cable length L equal to zero to obtain the optimum delay-line length and phase-noise reduction, where $f(\,)$ and α are the self-injection signal strength function of the attenuation loss and attenuation loss parameter in delay-line cables, respectively. However, one also needs to make sure that the phase delay related to the cable length is within the stable loop phase-delay range in [14, Fig. 3]. If the delay-line cable is too long, the self-injection signal strength will be very small, and the self-injection-locked technique on oscillator phase-noise reduction will not take into effects.

The spectral characteristics of the phase noise in a self-injection-locked parallel-resonant oscillator with a feedback delay loop are shown in Fig. 1 for illustration. The phase noise of the oscillator follows the ideal $1/f^2$ dependence. As the self-injection signal strength ρ increases, the phase noise is reduced further. For a longer feedback delay T while keeping $\cos\Delta\hat{\theta} > 0$, the phase noise is reduced further.

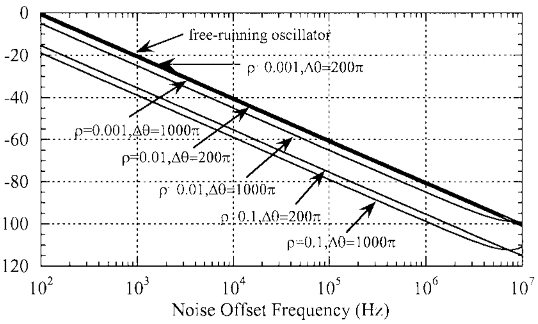

Fig. 1. Simulated phase noise of the self-injection-locked parallel resonant oscillator with a delay cable of $\Delta\theta = 200\pi$ (the delay $T = 100/f_0$) and $\Delta\theta = 1000\pi$ (the delay $T = 500/f_0$) in the loop, where $f_0 = 8.0\ GHz$ is the frequency of the self-injection-locked oscillator. The loop phase satisfies the stability condition (i.e., $\cos\Delta\hat{\theta}_p > 0$). As the self-injection signal strength ρ increases, the phase noise is reduced further for noise offset frequency near the carrier frequency.

V. High-Q Factor Resonator in Feedback Loop

The high-Q factor resonator has been used to stabilize the oscillator signals and reduce the phase noise inside the oscillator circuits. If one extracts a part of the oscillator output signal passed through a high-Q factor resonator, one can get the injection signal with pure spectrum and reduce the phase fluctuation of the self-injection signal. The frequency response (or the transfer function) of the high-Q factor resonator can be written as

$$H(\omega) = \frac{\jmath\omega\left(\dfrac{\omega_r}{Q_r}\right)}{(\omega_r^2 - \omega^2) + \jmath\omega\left(\dfrac{\omega_r}{Q_r}\right)} \tag{18}$$

where ω_r and Q_r are the resonance frequency and Q factor of the resonator, respectively, and (ω_r/Q_r) is the entire 3-dB bandwidth of the resonator.

For the oscillator with a self-injection signal passed through a high-Q factor resonator, the phase noise from (11) becomes

$$\begin{aligned}\frac{\left|\widetilde{\delta\theta}(\omega_m)\right|^2}{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2} = &\left[1 + \left(\frac{(\omega_0-\omega_r)+\omega_m}{\dfrac{\omega_r}{2Q_r}}\right)^2\right] \\ &\cdot\left\{\left[1 + \left(\frac{(\omega_0-\omega_r)+\omega_m}{\dfrac{\omega_r}{2Q_r}}\right)^2\right]\right. \\ &+ \rho^2\left(\frac{\omega_{3\mathrm{dB}}}{\omega_m}\right)^2\cos^2\Delta\hat{\theta} \\ &\times\left(\frac{(\omega_0-\omega_r)+\omega_m}{\dfrac{\omega_r}{2Q_r}}\right)^2 \\ &\pm 2\rho\left(\frac{\omega_{3\mathrm{dB}}}{\omega_m}\right)\cos\Delta\hat{\theta} \\ &\left.\times\frac{(\omega_0-\omega_r)+\omega_m}{\left(\dfrac{\omega_r}{2Q_r}\right)}\right\}^{-1}\end{aligned} \tag{19}$$

where the upper sign is for parallel oscillators and the lower sign is for series oscillators, and (18) has been used and written as

$$\begin{aligned}H(\omega_0+\omega_m) &= \frac{\jmath(\omega_0+\omega_m)\dfrac{\omega_r}{Q_r}}{\left[\omega_r^2 - (\omega_0+\omega_m)^2\right] + \jmath(\omega_0+\omega_m)\dfrac{\omega_r}{Q_r}} \\ &\approx \frac{1}{1 + \jmath\dfrac{(\omega_0-\omega_r)+\omega_m}{\left(\dfrac{\omega_r}{2Q_r}\right)}}\end{aligned} \tag{20}$$

in order to get a clear insight into the phase-noise properties. Here, we are interested in the phase noise around ω_0 and $\omega_0 = \omega_r$, and the phase noise of the self-injection-locked oscillator becomes

$$\frac{\left|\widetilde{\delta\theta}(\omega_m)\right|^2}{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2} = \frac{1 + \left(\dfrac{\omega_m}{\omega_{3\mathrm{dB}}}\right)^2\left(\dfrac{Q_r}{Q}\right)^2}{\left(1 \pm \rho\cos\Delta\hat{\theta}\dfrac{Q_r}{Q}\right)^2 + \left(\dfrac{\omega_m}{\omega_{3\mathrm{dB}}}\right)^2\left(\dfrac{Q_r}{Q}\right)^2} \tag{21}$$

where the upper sign is for parallel oscillators and the lower sign is for series oscillators.

From the stability analysis, one can use $\Delta\hat{\theta} = 0$ (or 2π) in parallel oscillators and $\Delta\hat{\theta} = \pi$ in series oscillators for the optimum stable output phase, and the phase noise of the oscillator with the self-injection signal passed through a high-Q factor resonator is

$$\frac{\left|\widetilde{\delta\theta}(\omega_m)\right|^2}{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2} = \frac{1 + \left(\dfrac{\omega_m}{\omega_{3\mathrm{dB}}}\right)^2\left(\dfrac{Q_r}{Q}\right)^2}{\left(1 + \rho\dfrac{Q_r}{Q}\right)^2 + \left(\dfrac{\omega_m}{\omega_{3\mathrm{dB}}}\right)^2\left(\dfrac{Q_r}{Q}\right)^2}. \tag{22}$$

From the above equation, one can find that the phase noise is reduced in the self-injection-locked oscillator with a high-Q factor resonator in the loop. If $\rho = 0$, there is no self-injection signal, and the phase noise becomes its free-running value $|\widetilde{\delta\theta}_0(\omega_m)|^2$.

For the phase noise near the carrier frequency $(\omega_m \to 0)$

$$\lim_{\omega_m\to 0}\left|\widetilde{\delta\theta}(\omega_m)\right|^2 \to \frac{\left|\widetilde{\delta\theta}_0(\omega_m)\right|^2}{\left(1 + \rho\dfrac{Q_r}{Q}\right)^2} \tag{23}$$

One can find that the phase-noise reduction factor at the carrier frequency is decided by Q_r/Q, the Q factor ratio of the high-Q factor resonator to the oscillator. If the Q factor ratio increases, the phase noise is reduced further because the high-Q factor resonator is an excellent bandpass filter centered at $\omega_r = \omega_0$, which shapes the spectrum of the self-injection signal as a low phase-noise signal source. As the self-injection signal strength ρ increases, the phase noise is reduced further.

At another extreme (far from the carrier frequency $\omega_m \to \infty$), $|\widetilde{\delta\theta}(\omega_m)|^2 \to |\widetilde{\delta\theta}_0(\omega_m)|^2$. The phase noise returns to its free-running value and the self-injection locking has no effect on noise reduction. The results are the same as the phase-noise properties of the oscillator locked to an external low phase-noise signal [8].

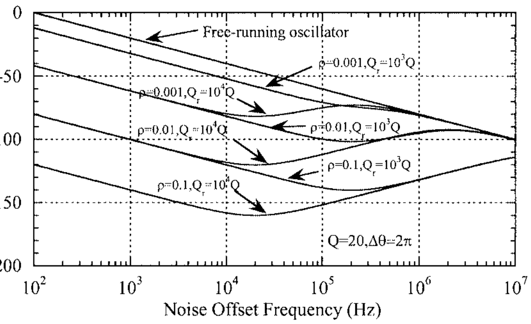

Fig. 2. Simulated phase noise of the self-injection-locked parallel-resonant oscillator with different self-injection signal strength ρ and high-Q factor resonator (Q_r) in the loop. The oscillator and resonance frequency of the resonator 8 GHz, the loop phase $\Delta\hat{\theta}_p = 2\pi$, and the Q factor of the oscillator $Q = 20$ are used in the simulations, and the loop phase satisfies the stability condition (i.e., $\cos\Delta\hat{\theta}_p > 0$). As the self-injection signal strength ρ or Q_r increases, the phase noise is reduced further for noise offset frequency near the carrier frequency. For noise offset frequency far from the carrier frequency, the phase noise is returned to its free-running value.

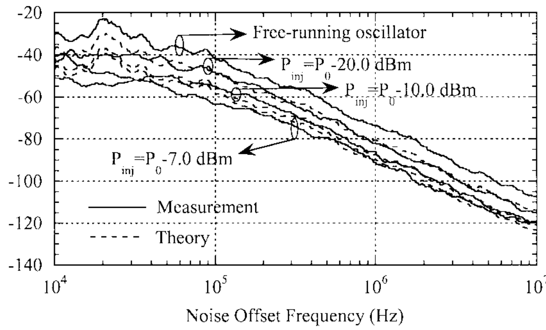

Fig. 3. Experimental results of free-running oscillator phase noise and phase noise of the self-injection-locked oscillator with the loop delay of 15.70 ns. The oscillator output power is $P_0 = 5.5$ dBm. We use different attenuators and measure the total insertion losses of the self-injection feedback loop, while keeping the loop under the stable condition (i.e., $\cos\Delta\hat{\theta}_p > 0$). The results show good qualitative agreement with theoretical values and Fig. 1. The phase-noise curves depart from the ideal curves within some noise offset frequency ranges, which may be caused by the change of the oscillator output load, or averaging and smoothing phase-noise measurements in ten successive spectrum sweeps in an Agilent E4407B phase-noise measurement personality. The phase noise is reduced as the self-injection signal strength increases for the self-injection locked oscillator.

If $\Delta\hat{\theta} = \pi$ for parallel oscillators and $\Delta\hat{\theta} = 0$ (or 2π) for series oscillators, from the stability analysis, the output oscillator phase is not stable, and the phase noise cannot be reduced under this condition.

If $\Delta\hat{\theta} = (2m+1)\pi/2$, where m is an integer, $|\widetilde{\delta\theta}(\omega_m)|^2 = |\widetilde{\delta\theta}_0(\omega_m)|^2$ and the self injection signal has no effect on phase-noise reduction at all for both parallel and series oscillators.

The phase-noise characteristics of the self-injection-locked oscillator with a high-Q factor resonator are shown in Fig. 2. Here, we use the parallel-resonant oscillator and the high-Q factor resonator with Q_r and resonance frequency $\omega_r = \omega_0$ and the loop delay phase $\Delta\hat{\theta} = 2\pi$ as illustrations. As the self-injection signal strength ρ increases, the phase noise is reduced further for the noise offset frequency near the carrier frequency. If the Q factor of the resonator is increased, the noise reduction effect is more evident near the carrier frequency.

VI. Experimental Results

An oscillator was used for the experimental verification of this paper's theory. The oscillator is a varactor-tuned MESFET voltage-controlled oscillator (VCO) with a nominal tuning range of 8–9 GHz. The VCO uses an NE32184A packaged MESFET and M/A-COM 46600 varactor diode, and is fabricated on a Rogers Duroid board 5880 ($\epsilon_r = 2.2$) with the thickness 0.787 mm [7], [8]. The output power of the oscillator is $P_0 = 5.5$ dBm. The Q factor of the oscillator can be decided from the injection-locking range [7], [8], and $Q = 17$ at $\omega_0 = 2\pi * 8.5$ GHz. The oscillator is parallel resonant, which is verified by the coupled oscillator array with an antenna array by testing its radiation patterns [13].

The measurement setup is the same as [14, Fig. 1 or 4], and one output port of the power divider is connected to the Agilent Spectrum Analyzer E4407B with phase-noise measurement personality (option 226) [5]. Time-domain reflectometer (TDR) is also used to test the delays of the individual components in the loop, including the cable connecting the oscillator and circulator, the delay within circulator ports, the connection between the circulator and power divider, the delay within power-divider ports, and the feedback cable between the power divider and circulator, and then add up those delays in the feedback loop as the total delay. One can put different attenuators or cable length in the feedback loop to change the self-injection signal strength and delay, respectively. We then observe the self-injection-locked oscillator output spectrum while keeping the whole setup to satisfy the stability conditions.

First, we want to test the phase noise of the free-running oscillator. We use the setup of the self-injection-locked oscillator, except there is no feedback injection signal. One output port of the power divider is connected to the spectrum analyzer, and the other output port is terminated with a 50-Ω load. The self-injection signal port of the circulator is also terminated with a 50-Ω load. The free-running frequency of the oscillator is 8.193 GHz, and the measured phase-noise result is shown in Fig. 3. When we apply the self-injection signal, there is a slight frequency shift from 8.193 to 8.150 GHz due to the change of the oscillator output load.

Here, we are interested in the phase-noise measurements on the loop delay of 15.70 ns. We use different attenuators and measure the total insertion losses of the self-injection feedback loop, while keeping the loop under the stable condition (i.e., $\cos\Delta\hat{\theta}_p > 0$). The phase-noise measurement results are also shown in Fig. 3. The results show good qualitative agreement with Fig. 1. The phase-noise curves depart from its ideal curves within some noise offset frequency ranges, which may be caused by the change of the oscillator output load or averaging and smoothing the phase-noise measurements in ten successive spectrum sweeps in an Agilent E4407B phase-noise measurement personality. We find the phase noise is reduced as the self-injection signal strength increases for the self-injection-locked oscillator. This confirms our oscillator phase-noise analysis of self-injection-locked oscillators.

VII. Conclusions

Phase-noise analysis of the self-injection-locked oscillator has been presented. The analysis has been developed for different oscillator models and arbitrary self-injection feedback loops. The results are illustrated with specific cases of a simple time-delay cable and a high-Q factor resonator. The behavior of the phase noise is similar to an oscillator locked to the external low phase-noise source. The phase noise is reduced at the noise offset frequency near the carrier frequency, and returning to the free-running oscillator noise far from the carrier frequency for certain stable feedback delay ranges. The phase-noise reduction is affected by the self-injection signal strength and feedback transfer function for different oscillator equivalent-circuit models and certain stable feedback delay ranges. For a high-Q factor resonator with resonance frequency centered at the carrier frequency in the loop, as the Q factor increases, the phase noise is reduced further because the high-Q factor resonator is an excellent bandpass filter and it shapes the self-injection signal as the low phase-noise signal source. The self-injection-locked technique may be used to improve the phase noise of the existing oscillators.

The theory is verified by using a self-injection-locked GaAs MESFET oscillator operating at the X-band with delay cable loops. The phase-noise properties for the stable self-injection-locked oscillator have similar performances as the oscillator locked to an external low phase-noise source. As the self-injection signal strength increases for stable loop phase conditions, the phase noise is reduced. There are still several aspects of the noise analysis not treated in this paper. The first one is the influence of amplitude noise and AM-to-PM noise conversion. Those effects are considered negligible for the noise offset frequency near the carrier frequency in this paper. Secondly, in the derivation of the phase noise in self-injection-locked oscillators, we use the assumption that the loop phase is constant with respect to time. However, there is still some small variation of the oscillator frequency due to the change of the oscillator output load. If the loop phase is not constant with respect to time, the phase-noise reduction factor for the self-injection-locked oscillator will not be fixed at the noise offset frequency ω_m relative to the carrier frequency ω_0. The effects of the oscillator frequency change and loop phase variation should be studied carefully in the future.

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